Turbomachinery & Turbulence. Lecture 2: One dimensional thermodynamics.

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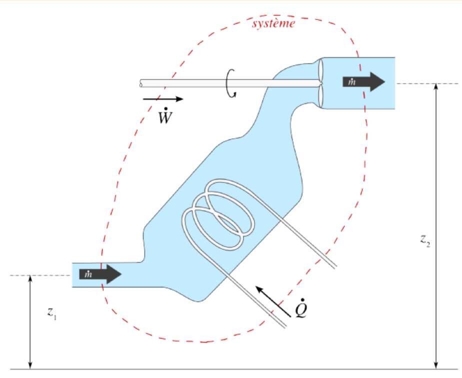
Global balance equations in open systems

Fixed control volume



- Mass balance
- Momentum balance
- Total Energy balance

Flow work, shaft work and enthalpy

- To push a volume $\dot{V}_{in}$ of fluid inside the control volume each second, the exterior gives to the system:

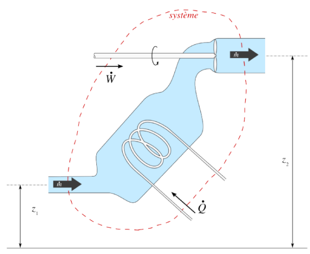
$$\dot{W}_{in} = p_{in}\dot{V}_{in} = \frac{p_{in}}{\rho_{in}}\dot{m}_{in}$$

That is a specific work

$$w_{in} = \frac{p_{in}}{\rho_{in}}$$

- To extract the fluid of the control volume, the system restitutes a specific work to the exterior:

$$w_{out} = -\frac{p_{out}}{\rho_{out}}$$



- The "specific flow work" $\frac{p}{\rho}$ is included into the specific *enthalpy* $h = \tilde{u} + \frac{p}{\rho}$
- For an open-system, under steady conditions:

$$\dot{W} + \dot{Q} = \dot{m}\Delta\left(h + \frac{C^2}{2} + gz\right)$$

Global balance equations in open systems

- Mass balance

$$\frac{\partial\left(\rho V\right)}{\partial t}=\dot{m}_{in}-\dot{m}_{out}$$

- Momentum balance

$$\iiint_V \frac{\partial\left(\rho\vec{C}\right)}{\partial t}dv+\oiint_S \rho\vec{C}\left(\vec{C}\cdot\vec{n}\right)ds \quad = \quad \iiint_V \rho\vec{f}dv+\oiint_S \overline{\overline{\tau}}\cdot\vec{n}ds$$

- Energy balance

$$\iiint_V \frac{\partial}{\partial t}\left[\rho\left(\frac{C^2}{2}+gz+\tilde{u}\right)\right]dv+\oiint_S \rho\left[\frac{C^2}{2}+gz+\tilde{u}+\frac{p}{\rho}\right]\left(\vec{C}\cdot\vec{n}\right)ds \quad = \quad \dot{W}+\dot{Q}$$

Exercice 1

- The compressor of a turboreactor takes $\dot{m} = 1.5$ kg.s^{-1} of air at 0.8 bar with an internal energy $\tilde{u} = 192.5$ kJ.kg^{-1} and a specific volume of 0.96 m^3.kg^{-1}.
- The air is compressed at 30 bar and has then an internal energy $\tilde{u} = 542.3$ kJ.kg^{-1} and a specific volume of 6.19×10^{-2} m^3.kg^{-1}.
- There is no change in velocity and no heat transfer.
- What is the power of the compressor?

Exercice 2

- In a nozzle, air is expanded without work or heat transfer.
- The air enters with a specific enthalpy $h = 776$ kJ.kg^{-1} and a speed of 30 km/h.
- It leaves the nozzle with a specific enthalpy $h = 636$ kJ.kg^{-1}.
- What is the outlet speed?

Mechanical energy and efficiency

- Turbomachines are used to vehiculate fluids, to compress them, or to recover energy.
- They imply generation or consumption of mechanical energy.
- Mechanical energy: form of energy that can be converted to work completely and directly by an ideal machine.
- Specific mechanical energy of a fluid:

$$e_{mech} = \frac{p}{\rho} + \frac{C^2}{2} + gz$$

- *flow energy* + *kinetic energy* + *potential energy*
- it looks like the right-hand side of

$$\dot{W} + \dot{Q} = \dot{m}\Delta\left(h + \frac{C^2}{2} + gz\right)$$

  doesn't it?

Mechanical energy and efficiency

- Turbomachines are often operating adiabatically:
- If no "irreversible losses" are present:

$$\dot{W} = \dot{m}\Delta\left(e_{mech}\right)$$

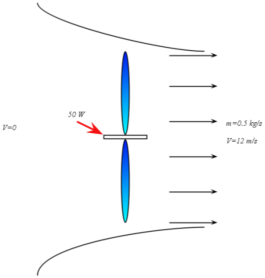
- "losses" are actually a degradation of *mechanical* energy into *thermal/internal* energy, *i.e.* **losses of mechanical energy**
- The mechanical efficiency of a process is:

$$\eta_{mech} = \frac{Mech.\ Energy\ Output}{Mech.\ Energy\ Input} = 1 - \frac{E_{mech.\ loss}}{E_{mech.\ in}}$$

- For a compression: $\eta = \frac{\Delta \dot{E}_{mech,fluid}}{\dot{W}_{shaft}}$
- For an expansion: $\eta = \frac{\dot{W}_{shaft}}{\Delta \dot{E}_{mech,fluid}}$

Exercice 1

- A fan absorbs 50 W to impulse 0.5 kg.s^{-1} of air from rest to $V = 12$ m.s^{-1}
- What is its efficiency?



Exercice 2

- The water in a lake is to be used to generate electricity.
- The elevation difference between the free surfaces upstream and downstream of the dam is 50 m.
- Water is to be supplied at a rate of 5000 kg.s^{-1}.
- The turbine has an efficiency of 80%.
- What is the power available on the shaft?

Gibbs equation

- Entropy s: in thermodynamics, a measure of the number of specific ways in which a thermodynamic system may be arranged.
- A state function: the change in the entropy is the same for any process that goes from a given initial state to a given final state (reversible or irreversible).
- For a reversible process:

$$ds = \frac{\delta q_{rev}}{T}$$

- First principle:

$$d\tilde{u} = \delta w + \delta q$$

- The internal energy $\tilde{u}$ is also a state function. For a reversible process:

$$\delta w_{rev} = -pd\left(\frac{1}{\rho}\right)$$

Thus:

$$d\tilde{u} = -pd\left(\frac{1}{\rho}\right) + Tds$$

$$dh = \frac{dp}{\rho} + Tds$$

Equation of state for a perfect gas

- $p = \rho r T$ ($r = 287$ J.kg^{-1}.K^{-1} for air).
- $\tilde{u} = c_v T$
- $h = c_p T$
- $r = c_p - c_v$
- $\gamma = \frac{c_p}{c_v}$ ($\gamma = 1.4$ for a diatomic gas).
- Show that $p = \rho\,(\gamma - 1)\,\tilde{u}$, compute c_p and c_v for air.

Isentropic transformations

- $p\rho^{-\gamma} = \mathrm{cte}$
- $T\rho^{1-\gamma} = \mathrm{cte}$
- $p^{\gamma-1}T^{-\gamma} = \mathrm{cte}$
- $\frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma}$
- $\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$
- Celerity of sound waves: $a^2 = \left(\frac{\partial p}{\partial \rho}\right)_s$. For a perfect gas: $a = \sqrt{\gamma r T}$.

Exercice 1

- A liquid pump rises the pressure of water from 1 to 20 bars.
- The mass-flow rate is 2 kg.s^{-1}.
- The specific volume is constant and is $v_L = 10^{-3}$ $\text{m}^3.\text{kg}^{-1}$.
- What is the power needed for an isentropic process?
- What is the temperature rise for an efficiency of 80%? ($c = 4.18$ $\text{kJ.kg}^{-1}.\text{K}^{-1}$).

Exercice 2

- A compressor rises the pressure of air from 1 to 20 bars.
- The mass-flow rate is 2 kg.s^{-1}.
- The process is isentropic: $p(1/\rho)^{1.35} = \text{cte}$. At the inlet the specific volume is $v_a = 0.8$ $\text{m}^3.\text{kg}^{-1}$.
- What is the power needed?
- What is the increase in temperature?

Stagnation enthalpy and temperature

- Stagnation (or total) enthalpy: fluid brought to rest with no heat or work transfer

$$h_0 = h + \frac{C^2}{2}$$

- For a perfect gas with $h = c_p T$, with constant c_p, one defines:

$$T_0 = T + \frac{C^2}{2c_p}$$

Isentropic stagnation pressure and density

- For a (fictious) isentropic transformation of a perfect gas:

$$\frac{p_0}{p} = \left(\frac{T_0}{T}\right)^{\frac{\gamma}{\gamma-1}} \quad \Rightarrow \quad p_0 = p\left(1 + \frac{C^2}{2c_p T}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{\rho_0}{\rho} = \left(\frac{T_0}{T}\right)^{\frac{1}{\gamma-1}} \quad \Rightarrow \quad \rho_0 = \rho\left(1 + \frac{C^2}{2c_p T}\right)^{\frac{1}{\gamma-1}}$$

Isentropic stagnation pressure and density

- For a perfect gas:

$$\frac{C^2}{2c_p T} = \frac{\gamma r T}{2c_p T} M^2 = \frac{\gamma - 1}{2} M^2$$

- For a (fictious) isentropic transformation of a perfect gas:

$$T_0 = T\left(1 + \frac{\gamma - 1}{2} M^2\right)$$

$$p_0 = p\left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma - 1}}$$

$$\rho_0 = \rho\left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{1}{\gamma - 1}}$$

Incompressible flow

- For $M = 0.3$, what is the relative variation of density?
- The flow is incompressible $M \leq 0.3$
- What becomes the relation between stagnation and static pressure in that case ?

Application to turbomachinery

- For an adiabatic situation:

$$dW_{shaft} = Tds + \frac{dp}{\rho} + d\frac{C^2}{2}$$

$$dW_{shaft} = T_0 ds_0 + \frac{dp_0}{\rho_0}$$

- $T_0 ds_0$ are dissipation of mechanical energy (losses).
- For a perfect gas:

$$\begin{aligned} W_{shaft} &= c_p \left(T_{02} - T_{01}\right) \\ &= c_p T_{01} \left(\frac{T_{02}}{T_{01}} - 1\right) \end{aligned}$$

- In case of an isentropic transformation:

$$W_{shaft} = c_p T_{01} \left(\left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma - 1}{\gamma}} - 1\right)$$

Thermodynamic diagrams

- Entropic T, s and Enthalpic h, s diagrams.
- For an adiabatic evolution in an open system:
  - Ordinates stand for the *energy* in enthalpic diagram.
  - Abscissa stand for the *degree of irreversibility*.
  - Losses are best shown in entropic diagram ($\int_i^f Tds$).
- Isentropic evolutions are vertical path.
- In real cases, the entropy rises (2^{nd} principle).
- $\Rightarrow$ Definition of efficiencies.

Efficiency

- Efficiency of a turbomachine is one of the most important performance parameters, but also one of the most ill-defined.
- Idea: to compare the actual work transfer to that which would occur in an ideal process.
- The ideal process: reversible between the same end states.
- But what are the "relevant"end states?
- And what is the reversible reference transformation?
  - Incompressible flow $\Rightarrow$ straightforward definition based on the fluid mechanical energy variation.
  - Compressible flow $\Rightarrow \int_i^f \frac{dp}{\rho}$ is path-dependent.

Polytropic transformation

- For an infinitesimal transform between two defined end states:

$$\delta w - \delta w_{rev} = \delta f > 0$$

- Polytropic evolution: the reversible evolution the fluid would experience following the same path as the actual transformation.
- Actual energy transfer W_{shaft}.
- Polytropic work $\tau_p = \int_i^f \frac{dp}{\rho} + \Delta\left(\frac{C^2}{2}\right)$.
- Internal losses:

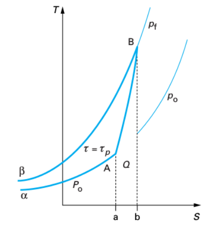
$$\Delta f = W_{shaft} - \tau_p$$

Polytropic coefficient

- The real path $\rho(p)$ is modeled:

$$p\rho^{-k} = \text{cte}$$

$$k = \frac{\ln\left(\frac{p_f}{p_i}\right)}{\ln\left(\frac{\rho_f}{\rho_i}\right)}$$



Polytropic efficiency for an ideal gas

$$\frac{k-1}{k} = \frac{\ln\left(\frac{T_{0,f}}{T_{0,i}}\right)}{\ln\left(\frac{p_{0,f}}{p_{0,i}}\right)}$$

$$\tau_p = \frac{k}{k-1} r \left(T_{0,f} - T_{0,i}\right)$$

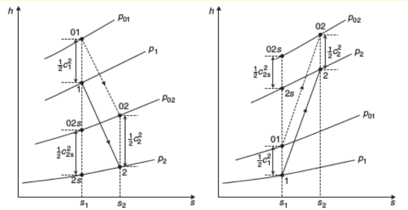
$$\eta_p = \frac{\frac{k}{k-1}}{\frac{\gamma}{\gamma-1}}$$

Isentropic transformation

A reversible adiabatic transform between the same end states. But which ones ?

1. Between the same static pressures and velocities $\Rightarrow$ *isentropic* efficiency η_{is}.
2. Between the same total (stagnation) pressures $\Rightarrow$ *total to total* efficiency η_{tt}.
3. A mix $\Rightarrow$ *total to static* efficiency η_{ts}.

Visualisation of efficiencies in a h-s diagram



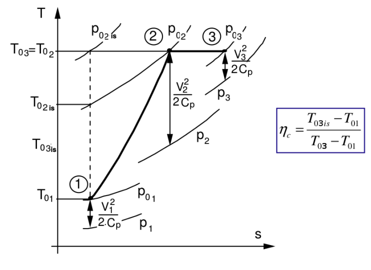
(a) Turbine expansion process (b) Compression process

The state $02s$ is not an actual state!

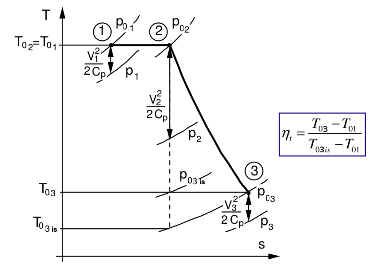
For an expansion process (steam and gas turbines):

1. $$\eta_{is} = \frac{h_2 - h_1 + \Delta\left(\frac{C^2}{2}\right)}{h_{2s} - h_1 + \Delta\left(\frac{C^2}{2}\right)}$$
2. exhaust kinetic energy wasted: $\eta_{ts} = \frac{h_{02} - h_{01}}{h_{2s} - h_{01}}$
3. exhaust kinetic energy usefull: $\eta_{tt} = \frac{h_{02} - h_{01}}{h_{02s} - h_{01}}$

Total to total efficiency of a *stage* in a T-s diagram



Compression stage



Expansion stage

- Compressible flow of an ideal gas: actual work measured with total temperature change.
- For a stator, $\Delta h_0 = 0 \Rightarrow \eta_{tt}$ of a stator has no meaning!
- Compression: the dissipation is linked to $p_{02,s} - p_{02}$ for the rotor and to $p_{03} - p_{02}$ for the stator.

Energy balance for a fluid particle

The time derivative of the kinetic energy and of the internal energy of a *fluid particle* is equal to the sum of the external forces power and the heat power:

$$\frac{d}{dt}\left(E_k + E_{int}\right) = \mathcal{P}_{ext} + \mathcal{P}_{cal}$$

$$E_k = \iiint_V \frac{1}{2}\rho C^2 \qquad \mathcal{P}_{ext} = \iiint_V \rho\vec{g}\cdot\vec{C} + \oiint_S \left(\overline{\overline{\sigma}}\cdot\vec{n}\right)\cdot\vec{C}$$

$$E_i = \iiint_V \rho\tilde{u} \qquad \mathcal{P}_{cal} = \iiint_V \rho r_e + \oiint_S \lambda\vec{\nabla}T\cdot\vec{n}$$

Local energy balance equation

$$\frac{d}{dt}\left(\rho\left(\tilde{u}+\frac{1}{2}\rho C^2\right)\right) + \rho\left(\tilde{u}+\frac{1}{2}\rho C^2\right)\mathrm{div}\vec{C} = \rho\vec{g}\cdot\vec{C} + \rho r_e + \mathrm{div}\left(\lambda\vec{\nabla}T\right) + \mathrm{div}\left(\overline{\overline{\sigma}}\cdot\vec{C}\right)$$

Momentum balance equation $(\cdot\vec{C})$ + mass balance equation $\Rightarrow$ Kinetic energy balance equation. By substraction:

$$\frac{\partial}{\partial t}\tilde{u} + \vec{C}\cdot\vec{\nabla}\tilde{u} = \frac{1}{\rho}\overline{\overline{\sigma}} : \overline{\overline{D}} + r_e + \frac{1}{\rho}\mathrm{div}\left(\lambda\vec{\nabla}T\right)$$

Stress-shear equation for a newtonian fluid

$$\overline{\overline{\sigma}} = \left(-p + \zeta \mathrm{div}\vec{C}\right)\overline{\overline{1}} + 2\mu\overline{\overline{D}}$$

$$\overline{\overline{D}} = \frac{1}{2}\left[\left(\vec{\nabla}\vec{C}\right) + {}^{t}\left(\vec{\nabla}\vec{C}\right)\right]$$

Local entropy balance equation

With the relation $d\tilde{u} = Tds + pd\left(\frac{1}{\rho}\right)$:

$$\rho T\frac{ds}{dt} = \rho r_e + \mathrm{div}\left(\lambda\vec{\nabla}T\right) + \mathrm{Tr}\left(2\mu\overline{\overline{D}}\cdot\overline{\overline{D}} + \zeta\mathrm{div}(\vec{C})\overline{\overline{1}}\cdot\overline{\overline{D}}\right)$$

Viscous dissipation term (newtonian incompressible fluid), in cylindrical coordinates:

$$\mathrm{Tr}\left(2\mu\overline{\overline{D}}\cdot\overline{\overline{D}}\right) = 2\mu\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right]$$

$$+\mu\left[\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)^2\right]$$