Turbomachinery & Turbulence. Lecture 4: Design and analysis of an axial-flow compression stage.

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Moment of momentum

- Fluid enters at a flow rate $\dot{m}$ at r_1 with tangential velocity $C_{\theta 1}$.
- It leaves the control volume at r_2 with tangential velocity $C_{\theta 2}$.
- Moment of momentum, steady version, along a streamline:

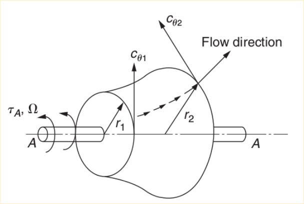
$$\tau_a = \dot{m}\left(r_2 C_{\theta 2} - r_1 C_{\theta 1}\right)$$

- Power:

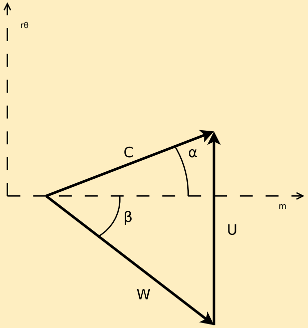
$$\tau_a \omega = \dot{m}\left(U_2 C_{\theta 2} - U_1 C_{\theta 1}\right)$$

- Link to energy exchange (steady process, adiabatic):

$$\Delta h_0 = \Delta\left(U C_\theta\right)$$



Rothalpy



- Along a streamline, the quantity called *rothalpy* is constant:

$$I = h_0 - UC_\theta = \mathrm{cte}$$

- Using the velocity triangle:

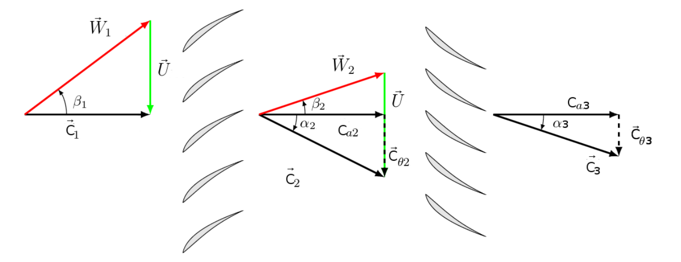
$$I = h + \frac{W^2}{2} - \frac{U^2}{2} = \mathrm{cte}$$

- Different contributions:

$$\Delta h_0 = \Delta\left(UW_\theta\right) + \Delta\left(U^2\right)$$

- *Aerodynamic forces work + Coriolis forces ($2\vec{\omega} \times \vec{W}$) work.*

Axial-flow compression stage

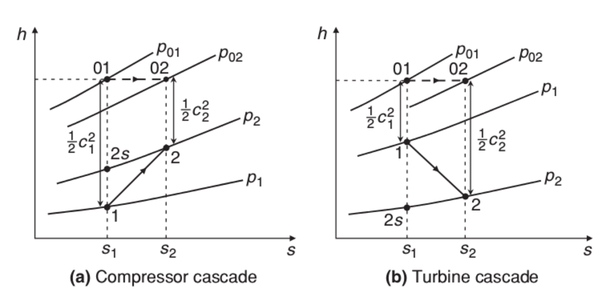


- Axial-flow stage: $U = \text{cte}$ along a streamline. If one assumes $C_m = \text{cte}$:

$$
\begin{aligned}
\Delta h_0 &= U\left(W_{\theta 2} - W_{\theta 1}\right) \\
&= UC_m\left(\tan\beta_2 - \tan\beta_1\right)
\end{aligned}
$$

- Watch out: $\beta < 0$. For a compression, $|W_{\theta 2}| < |W_{\theta 1}| \Rightarrow h_2 > h_1$.
- Stator: $h_0 = \text{cte}$ but $|C_3| < |C_2| \Rightarrow h_3 > h_2$.
- Conversion of kinetic energy to pressure (and degradation to internal energy).
- The relative flow is decelerated in the rotor. The absolute flow is decelerated in the stator. Diffusion (adverse pressure gradient) limits deflection to 40^o.

Mechanical energy loss



Mollier diagram for a steady blade cascade $h_{02} = h_{01}$.

Losses are related to Δp_0:

$$\int T_0 ds_0 = -\int \frac{dp_0}{\rho_0}$$

$$\Delta f \simeq \frac{p_{02} - p_{01}}{\rho_{01}}$$

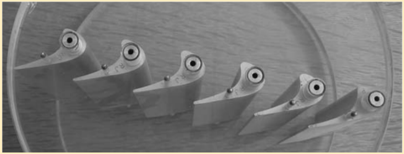
Losses with respect to isentropic are related to kinetic energy:

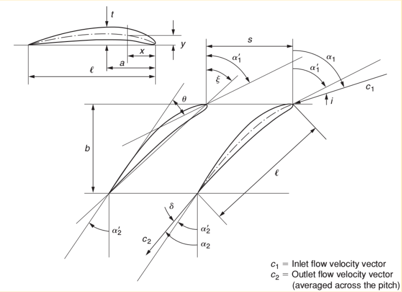
$$\text{Is. Loss} = \frac{1}{2}\left(C_{2s}^2 - C_2^2\right)$$

Blade cascade

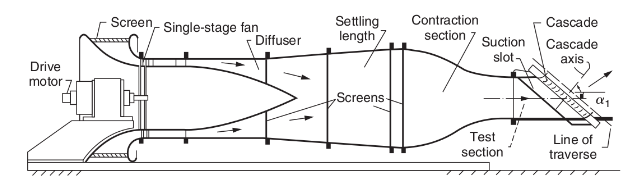
$$\Delta h_0 = UC_m (\tan\beta_2 - \tan\beta_1)$$

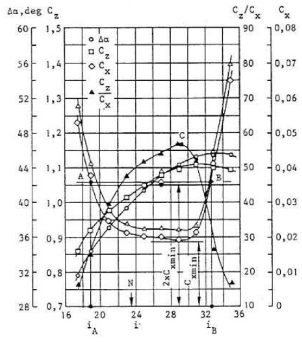
- Work depends on flow deflection $\Delta\beta$.
- Two-dimensional profile:
  - l: chord length
  - t: thickness of the profile
  - θ: camber angle.
  - $\alpha'_{1,2}$: blade inlet (outlet) angle
- Two-dimensional cascade:
  - ξ: stagger angle
  - $\sigma = l/s$: solidity
  - $\alpha_{1,2}$: inlet (outlet) flow angle
  - $i = \alpha_1 - \alpha'_1$: incidence angle
  - $\delta = \alpha_2 - \alpha'_2$: deviation
  - $\epsilon = \alpha_2 - \alpha_1$: deflection
  - $\text{aoa} = \alpha_1 - \xi$: angle of attack





Cascade characteristics

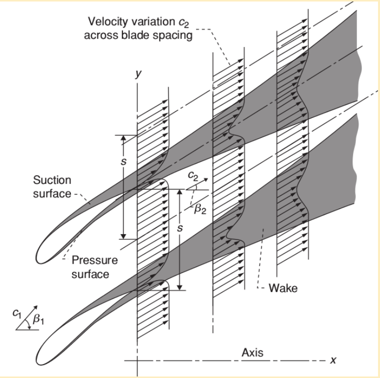




- Blade profiles: a certain thickness distribution (NACA65, British C series,...)
- Cascade characteristics: for given profiles, stagger angle and solidity,
- as a function of α_1, M_1, Re_1:
  - Exit flow angle α_2
  - stagnation pressure loss coefficient Y_P

The results are also presented as ϵ as a function of aoa, as lift and drag coefficient or as energy loss coefficient ζ.

Losses



- Stagnation pressure loss coefficient (compression):

$$Y_p = \frac{p_{01} - p_{02}}{p_{01} - p_1}$$

- Energy loss coefficient:

$$\zeta = \frac{\left(C_{2s}^2 - C_2^2\right)}{C_1^2}$$

- $Y_p = \zeta$ for $M \to 0$.

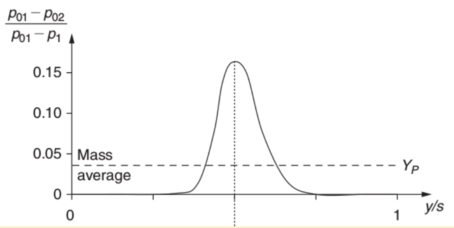
Cascade parameters measurements

- Measurements are performed $\simeq 1l$ upstream and downstream of the cascade.
- Mass-averaged quantities along one (two) pitch are given:

$$\dot{m} = \int_0^s \rho C_x \, dy$$

$$\tan \alpha_2 = \frac{\int_0^s \rho C_x C_y \, dy}{\int_0^s \rho C_x^2 \, dy}$$

$$Y_p = \frac{\int_0^s \left\{ (p_{01} - p_{02}) / (p_{01} - p_1) \right\} \rho C_x \, dy}{\int_0^s \rho C_x \, dy}$$



Application to rotors and stators

- Cascades are stationary ⇒ straightforward for stator blades.
- For rotors, replace:
  - α by β.
  - $\vec{C}$ by $\vec{W}$.
  - h_0 by $h_{0,rel}$.

Losses and efficiency: incompressible flow compression stage

- Incompressible flow, temperature change is negligible, $\rho = \mathrm{cte}$.
- Upstream rotor: 1, between rotor and stator: 2, downstream stator: 3.
- Actual work:

$$\Delta W = h_{03} - h_{01}$$

- Minimum work required to attain same final stagnation pressure:

$$\Delta W_{min} = h_{03ss} - h_{01}$$

- Along $p = p_{03}$, second law gives:

$$\Delta W_{min} = \Delta W - T \Delta s_{stage}$$

$$\eta_{tt} = \frac{\Delta W_{min}}{\Delta W} = 1 - \frac{T \Delta s_{stage}}{h_{03} - h_{01}}$$

Losses and efficiency: incompressible flow compression stage

- Accross the rotor, $h_{0,rel} = \mathrm{cte}$:

$$T\Delta s_{rotor} = \frac{\Delta p_{0,rel}}{\rho} = \frac{1}{2} W_1^2 Y_{p,rotor}$$

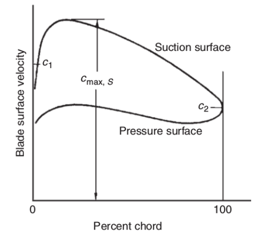
- Accross the stator, $h_0 = \mathrm{cte}$:

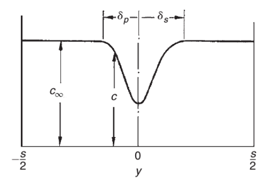
$$T\Delta s_{stator} = \frac{\Delta p_0}{\rho} = \frac{1}{2} C_2^2 Y_{p,stator}$$

- Thus:

$$\eta_{tt} = 1 - \frac{\frac{1}{2}\left(W_1^2 Y_{p,rotor} + C_2^2 Y_{p,stator}\right)}{h_{03} - h_{01}}$$

Losses and efficiency: incompressible flow compression stage



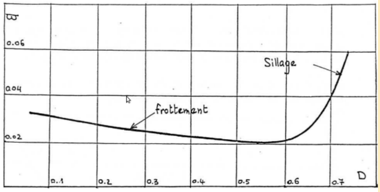


- Adverse pressure gradient: boundary layer growth (and detachment).
- Wake momentum thickness θ_2 correlated to diffusion on suction side.

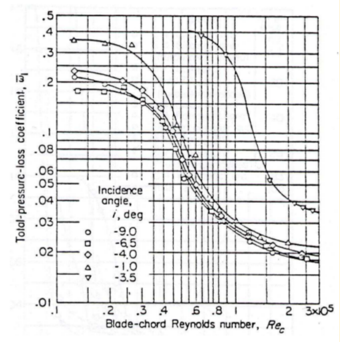
$$\theta_2 = \int_{-s/2}^{s/2} \left(\frac{C}{C_\infty} \right) \left(1 - \frac{C}{C_\infty} \right) dy$$

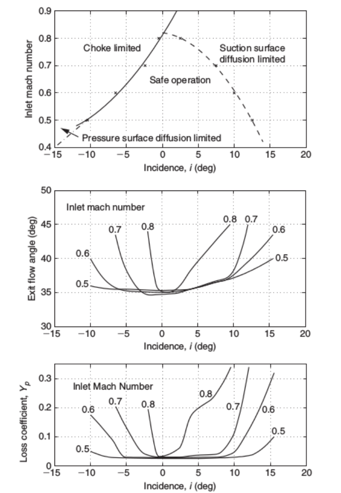
- Diffusion factor linked to solidity:

$$DF = \left(1 - \frac{C_2}{C_1} \right) + \left(\frac{|C_{\theta 2} - C_{\theta 1}|}{2\sigma C_1} \right)$$



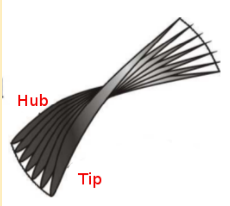
Effects of Reynolds number, Mach number and incidence





Radial dependence & spanwise velocities

1)



$U = r\omega \Rightarrow$ stagger angles depend on r.

2)

- Bernoulli theorem *accros* streamlines:

$$\frac{C_\theta^2}{r} = \frac{1}{\rho}\frac{\partial p}{\partial r}$$

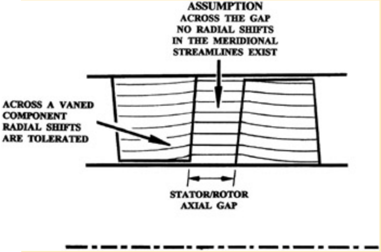
- For hub-to-tip ratio $r_h/r_t \lesssim 0.8$,
- temporary imbalance between centrifugal forces and radial pressure gradients.
- streamlines bend radially until sufficient radial transport to recover equilibrium.

3)

Simplified radial equilibrium hypothesis:

- Permanent flow;
- Outside blade rows;
- Cylindrical streamtubes;
- Viscous stress neglected:

Simplified radial equilibrium (incompressible flow)



$$\frac{C_\theta^2}{r} = \frac{1}{\rho}\frac{dp}{dr}$$

$$dh = Tds - \frac{dp}{\rho}$$

$$\frac{1}{\rho}\frac{dp_0}{dr} = \frac{dp}{dr} + C_\theta\frac{dC_\theta}{dr} + C_x\frac{dC_x}{dr}$$

$$\frac{1}{\rho}\frac{dp_0}{dr} = C_x\frac{dC_x}{dr} + \frac{C_\theta}{r}\frac{d}{dr}(rC_\theta)$$

Radial repartition of the work: vortex law

- Free vortex: $rC_\theta = \mathrm{cte}$
- Constant vortex: $C_\theta = \mathrm{cte}$
- Forced vortex: $C_\theta = \mathrm{cte} \cdot r$
- Constant absolute angle: $C_\theta / C_z = \mathrm{cte}$
- General vortex: $C_\theta = k_1 r^n + k_2 \frac{1}{r}$

"Passage vortex": mechanisms

Flows induced in transverse (S3) surfaces, by creation of meridional vorticity.

- Vorticity tends to conserve.
- Boundary layers on hub and casing are vortical regions (vorticity ω_p).
- Deflection ϵ.
- $\Rightarrow$ creation of a pair of passage vortices (ω_s).
- $\omega_s \simeq 2\epsilon\omega_p$.
- Other explanation based on blade-to-blade pressure gradient and streamline curvature.

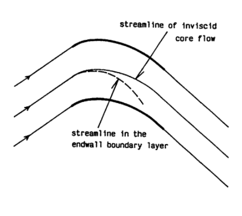
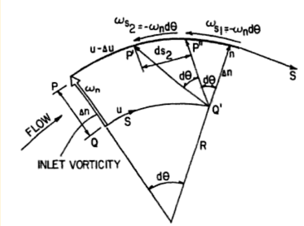
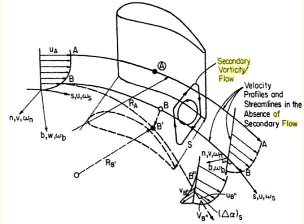


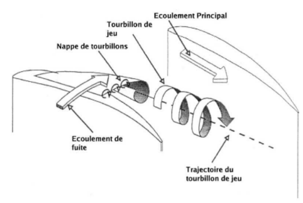
Fig. 2 Deflection of streamlines inside the endwall boundary layer





Other effects

- Horse-Shoe vortex: induced by boundary layer impinging on leading edge.



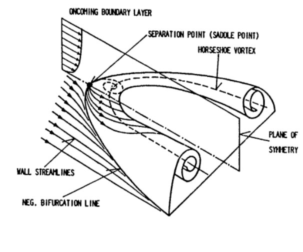


Fig. 1 Formation of a horseshoe vortex in front of an obstacle (obstacle removed)

- Blade boundary layers and wakes: low momentum fluid is centrifuged (radial secondary flow).
- Tip leakage vortex: induced by flow instability in the radial gap between blade and casing.

Secondary flows, a misleading term

- These mechanisms exist, but they all non-linearly interact:
- Extremely difficult to identify them.

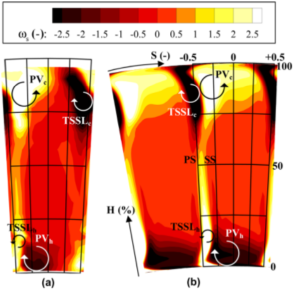


Fig. 5 Measured (5hp) (*a*) and predicted (*b*) streamwise vorticity ω_s at rotor inlet in the absolute frame of reference

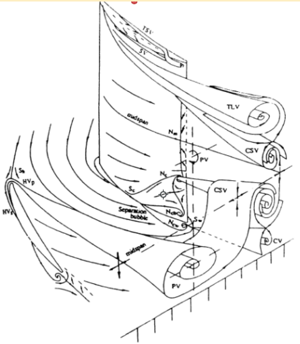
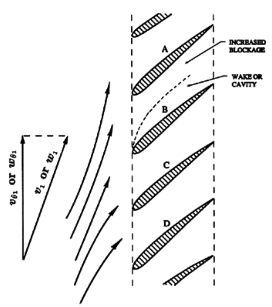


Figure 1. Schematic of secondary flow structure in a compressor cascade with tip clearance, Kang [6]: PV – Passage vortex; HV – Horseshoe vortex; TLV – Tip leakage vortex; TSV – Tip secondary vortex; SV – Secondary vortex; CSV – Concentrated shed vortex; CV – Corner vortex

Stall, Stage stall and surge



Rotating stall: frequency of the order of the rotating frequency.
Surge: system instability, slow time scales.

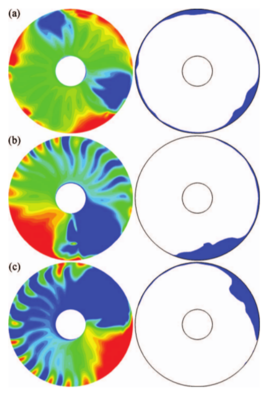


Fig. 7 Static pressure distribution (left) and reverse flow region (right) upstream of the fan in the rotating frame after 20 rev: (*a*) 70% fan speed, (*b*) 80% fan speed, and (*c*) 90% fan speed