Dimensional Analysis and Similarity
How to prepare a fluid flow experiment?

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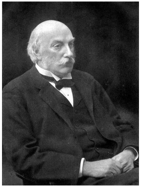
Introduction

"I have often been impressed by the scanty attention paid even by original workers in physics to the great principle of similitude. It happens not infrequently that results in the form of laws are put forward as novelties on the basics of elaborate experiments, which might have been predicted a priori after a few minutes' consideration".



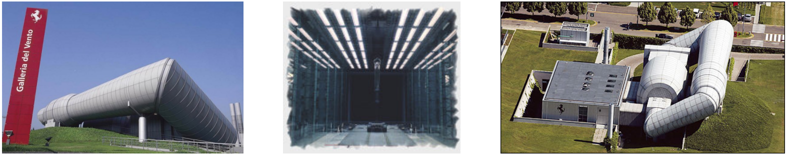
Lord Rayleigh, The principle of similitude, Nature,vol.XCV, No2368,pp.66-68,1915.

"We need to understand as soon as possible why the performance on track has not matched the figures coming out of the wind tunnel"
Stefano Domenicalli,
Scuderia Ferrari
Technical director.

150° Italia Ferrari

The 150° Italia was lagged aerodynamically due to a problem with the calibration of Ferrari's wind tunnel in 2011.

Reason: Scale changing (60% instead of 50%)



Maranello Wind Tunnel

Experiments: how to design a model in order to validate results and to understand what happens at full-scale ?

Dynfluid
Fluid Dynamics Laboratory

ARTS ET MÉTIERS ParisTech
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Why is **dimensional analysis** crucial for setting up experiments?

- Dimensionless numbers to generalize results
- Basic equations simplification (balance equations in fluid mechanics) by identification of negligible terms
- Reduction of relevant parameters needed for an experimental study (but also theoretical or numerical works)
- Determination of criteria to respect for model validation

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Dimensional analysis applications

Scale 1/200

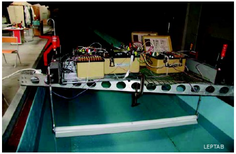
Scale 1/12

Scale 1/150

Dimensional analysis applications

Scale 1/11

Scale 1/15



Scale 1/20

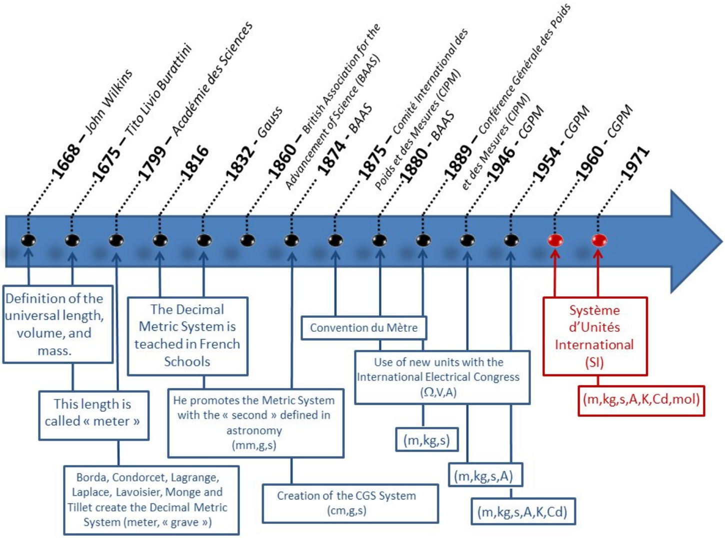
Outline

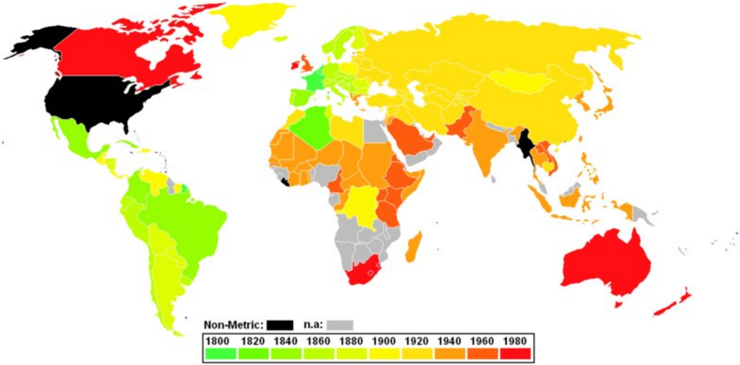
What is "International System of Units" ?

Because of the importance of a set of well defined and easily accessible units universally agreed for the multitude of measurements that support today's complex society, units should be chosen so that they are readily available to all, are constant throughout time and space, and are easy to realize with high accuracy.

- **International System of Units (SI)**
- **7 base units**

SI = Modern metric system of measurement established in 1960 by the 11th General Conference on Weights and Measures (CGPM, *Conférence Générale des Poids et Mesures)*
CGPM = intergovernmental treaty organization created by a diplomatic treaty called the Meter Convention (51 Member States)


1668 – John Wilkins
1675 – Tito Livio Burattini
1799 – Académie des Sciences
1816
1832 - Gauss
1860 – British Association for the Advancement of Science (BAAS)
1874 - BAAS
1875 – Comité International des Poids et des Mesures (CIPM)
1880 - BAAS
1889 – Conférence Générale des Poids et des Mesures (CIPM)
1946 - CGPM
1954 - CGPM
1960 - CGPM
1971
Definition of the universal length, volume, and mass.
This length is called « meter »
Borda, Condorcet, Lagrange, Laplace, Lavoisier, Monge and Tillet create the Decimal Metric System (meter, « grave »)
The Decimal Metric System is teached in French Schools
He promotes the Metric System with the « second » defined in astronomy (mm,g,s)
Creation of the CGS System (cm,g,s)
Convention du Mètre
Use of new units with the International Electrical Congress (Ω,V,A)
(m,kg,s)
(m,kg,s,A)
(m,kg,s,A,K,Cd)
Système d'Unités International (SI)
(m,kg,s,A,K,Cd,mol)



Use of the SI System around the world

What are the "7 base units"?

Base quantity	SI base unit name	SI base unit symbol	Dimension
Length	meter	m	L
Mass	kilogram	kg	M
Time	second	s	T
Electric current	ampere	A	I
Temperature	Kelvin	K	Θ
Amount of substance	mole	mole	N
Luminous intensity	candela	cd	J

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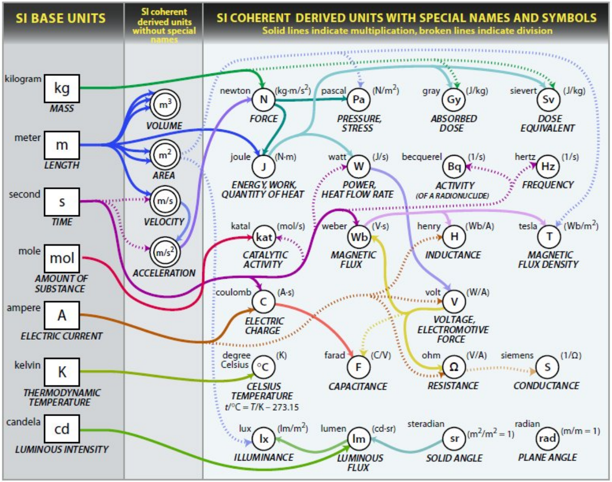
What happens to other quantities ?

- All other quantities are **derived quantities**, which may be written in terms of the base quantities by the equations of physics.
- The dimensions of the derived quantities are written as **products of powers of the dimensions of the base quantities** using the equations that relate the derived quantities to the base quantities

Derived quantities

Example: $v = \frac{\Delta x}{\Delta t} \Rightarrow [v] = \frac{L}{T}$



Inhomogeneous result is necessarily wrong!

...on the other side, homogeneous result is not perforce good...

Homogeneity rules

- Only homogenous terms may be added.
- The argument of a transcendental mathematical function (exp, ln, sin, cos, tan) is necessarily dimensionless.
- A vector is added only to a vector (never a scalar !).

A good method to remember units and to validate results !!

Examples: Dynamic viscosity μ unit ?

Kinematic viscosity ν unit ?

Dynamic viscosity μ

$\mu =$ ratio between shear stress and velocity gradient perpendicular to shear plane

$$\mu = \left(\frac{F}{A}\right) / \left(\frac{\mathrm{d}v}{\mathrm{d}y}\right)$$

Where: F: Applied tangential force $\Rightarrow [F] = M.L.T^{-2}$

A: Section $\Rightarrow [A] = L^2$

v: Velocity $\Rightarrow [v] = L.T^1$

y: Distance $\Rightarrow [y] = L$

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Kinematic viscosity ν

$$\nu = \frac{\mu}{\rho}$$

Where: ρ: Density $\Rightarrow [\rho] = M.L^{-3}$

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$$\nu = \frac{\mu}{\rho}$$

Where: ρ: Density $\Rightarrow [\rho] = M.L^{-3}$

Finally, $[\nu] = L^2.T^{-1} \Rightarrow$ Unit: $m^2.s^{-1}$

Outline

Dimensional Analysis = a method to **find empirical laws** when a theoretical resolution is too complex.

Let us consider a relation written as: $w = f(x, y, z)$

The quantity w is a function of dimensional quantities x, y and z (supposed to be independent).

The relation can be expressed as:

$$w = K(\ldots)x^{\alpha}y^{\beta}z^{\gamma}$$

K is a dimensionless number and only depends on other dimensionless numbers (e.g. $C_x = f(Re)$)

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How many relevant dimensionless numbers?

Vaschy-Buckingham theorem

Aimé Vaschy
(1857-1899)

If there are n **variables** in a problem and these variables contain k **primary dimensions** (for example M, L, T), the equation relating all the variables will have **(n-k) dimensionless products.**

Edgar Buckingham
(1867-1940)

Considering a physical phenomenon described by the law:

$$f\left(q_1, q_2, \ldots, q_n\right) = 0$$

where q_1,q_2,...,q_n are n independant parameters.

Then:

$$\phi\left(\pi_1, \pi_2, \ldots, \pi_{n-k}\right) = 0$$

Where π_i: are dimensionless products defined from q_i parameters.
In Fluid Mechanics $\Rightarrow$ $k = 3$ (if temperature effects are negligible, otherwise $k = 4$).
"**Never make a calculation until you know the answer**". *Wheeler, John A. and Edwin F. Taylor. Spacetime Physics, Freeman, 1966. Page 60.*

Dimensionless numbers deduction

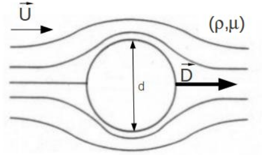
Method

1. Choice of k variables among the n parameters q_i (for example: q_1,q_2,...,q_k)
2. Creation of the $n-k$ groups:

$$
\begin{aligned}
\pi_1 &= q_1^{\alpha_1} q_2^{\alpha_2} \dots q_k^{\alpha_k} q_{k+1} \\
\pi_2 &= q_1^{\beta_1} q_2^{\beta_2} \dots q_k^{\beta_k} q_{k+2} \\
&\vdots \\
\pi_{n-k} &= q_1^{\eta_1} q_2^{\eta_2} \dots q_k^{\eta_k} q_{k+n}
\end{aligned}
$$

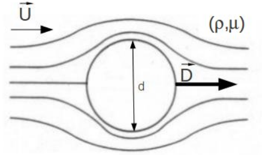
3. Calculation of exponents values by writing each quantity with base units and resolution of the k equation linear system for each exponent set $\{\alpha_1, \alpha_2, \dots, \alpha_k\}$, ..., $\{\eta_1, \eta_2, \dots, \eta_3\}$.

Some applications of the Vaschy-Buckingham theorem
The drag force on a sphere



- d: sphere diameter
- U: characteristic velocity of the uniform flow
- D: Drag force on the sphere
- ρ: fluid density
- μ: viscosity

Some applications of the Vaschy-Buckingham theorem
The drag force on a sphere



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$\Rightarrow n = 5$

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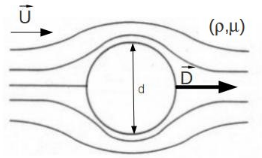

$$
\begin{aligned}
[d] &= L \\
[U] &= LT^{-1} \\
[D] &= MLT^{-2} \\
[\rho] &= ML^{-3} \\
[\mu] &= ML^{-1}T^{-1}
\end{aligned}
$$

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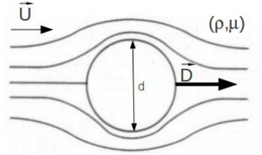

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$\Rightarrow k = 3$

$\Rightarrow n - k = 2$ **independent dimensionless numbers**

Method

1. Choice of 3 parameters:

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Method

1. Choice of 3 parameters: ρ, U, d
2. Creation of the 2 groups:

$$\begin{aligned}\pi_1 &= \rho^{\alpha_1} U^{\alpha_2} d^{\alpha_3} \mu \\ \pi_2 &= \rho^{\beta_1} U^{\beta_2} d^{\beta_3} D\end{aligned}$$

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Then:

$$\begin{aligned}\alpha_1 + 1 &= 0 \\ -3\alpha_1 + \alpha_2 + \alpha_3 - 1 &= 0 \\ -\alpha_2 - 1 &= 0\end{aligned}$$

And:

$$\begin{aligned}\beta_1 + 1 &= 0 \\ -3\beta_1 + \beta_2 + \beta_3 + 1 &= 0 \\ -\beta_2 - 2 &= 0\end{aligned}$$

so,

$$\begin{aligned}\alpha_1 &= -1 & \beta_1 &= -1 \\ \alpha_2 &= -1 & \beta_2 &= -2 \\ \alpha_3 &= -1 & \beta_3 &= -2\end{aligned}$$

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$$\pi_1 = \frac{\mu}{\rho U d} = \frac{1}{Re}$$

$$\pi_2 = \frac{D}{\rho d^2 U^2} = \frac{\pi}{8} C_D$$

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$$C_D = f\,(Re)$$

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Reynolds number

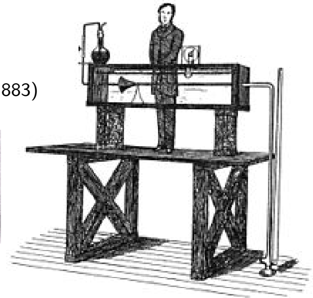
= measure of the ratio of inertial forces to viscous forces

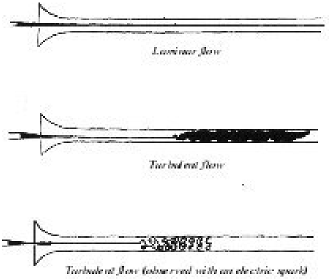
G.G. Stokes (1851)

$$Re = \frac{\rho U d}{\mu} = \frac{U d}{\nu}$$

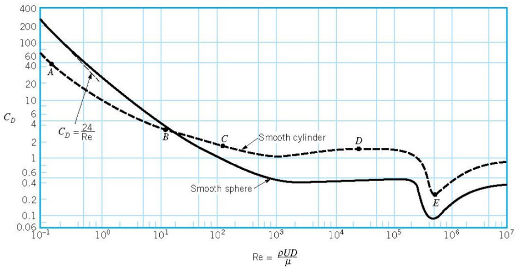
- ρ: Fluid density
- U: Fluid velocity
- d: Characteristic length
- μ: Dynamic viscosity
- ν: Kinematic viscosity

O. Reynolds (1883)





$$C_D = \frac{D}{\frac{1}{2}\rho U^2 \frac{\pi d^2}{4}}$$



C_D: Drag coefficient
Re: Reynolds number

- For $Re \ll 1$: $C_D = \frac{24}{Re}$ (Stokes)
- For $0.1 < Re < 5$: $C_D = \frac{24}{Re}\left(1 + \frac{3}{16}Re\right)$ (Lamb and Oseen)
- For $5 < Re < 800$: $C_D = \frac{24}{Re}\left(1 + 0.15 Re^{0.687}\right)$ (Schiller-Naumann)
- For $5 < Re < 400\ 000$: $C_D = \frac{24}{Re} + \frac{6}{1+\sqrt{Re}} + 0.4$ (White)

The drag force on a golf ball

The drag force on a golf ball

$\Rightarrow$ **n=6**: d, U, D, ρ, μ, ϵ.

The drag force on a golf ball

$\Rightarrow$ **n=6**: d, U, D, ρ, μ, ϵ.
$[D] = MLT^{-2}$
$[d] = L$
$[U] = LT^{-1}$
$[\rho] = ML^{-3}$
$[\mu] = ML^{-1}T^{-1}$
$[\epsilon] = L$
$\Rightarrow$ **k=3**: M, L, T
$\Rightarrow$ **n-k=3 independent dimensionless numbers**

$$C_D = f\left(Re, \frac{\epsilon}{d}\right)$$

Method

1. Choice of 3 parameters: ρ, U, d
2. Creation of the 3 groups:

$$\begin{aligned}\pi_1 &= \rho^{\alpha_1} U^{\alpha_2} d^{\alpha_3} \mu\\ \pi_2 &= \rho^{\beta_1} U^{\beta_2} d^{\beta_3} D\\ \pi_3 &= \rho^{\gamma_1} U^{\gamma_2} d^{\gamma_3} \epsilon\end{aligned}$$

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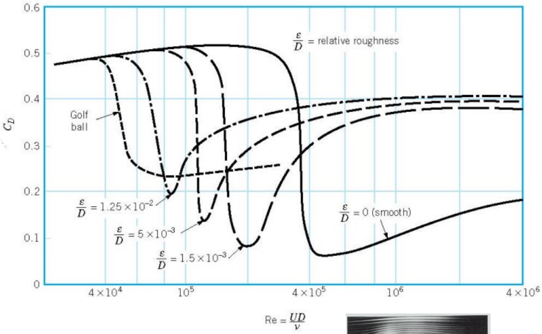
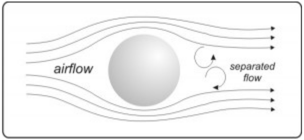
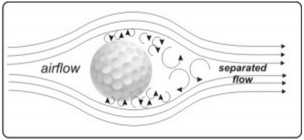
Reynolds Number

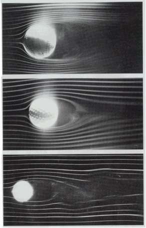
$$\pi_2 = \frac{D}{\rho d^2 U^2} = \frac{\pi}{8} C_D$$
Drag coefficient

$$\pi_3 = \frac{\epsilon}{d}$$
Relative roughness









Baseball

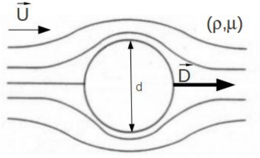
Golf ball

tennis ball

Order of magnitude of balance equations terms

Estimation of balance equations terms permits to:

1. neglect unimportant terms $\Rightarrow$ Problem simplification
2. obtain information about the solution before solving the problem



Momentum conservation equation (Navier-Stokes equation):

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}.\nabla\mathbf{v}\right) = -\nabla p + \rho\mathbf{g} + \nabla.\tau$$

(1) (2) (3) (4) (5)

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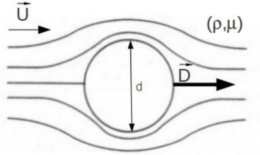
2. obtain information about the solution before solving the problem



(1): non-stationary acceleration per unit volume
(2): convective acceleration per unit volume
(3): pressure forces per unit volume
(4): body forces per unit volume
(5): viscous forces per unit volume

Momentum conservation equation (Navier-Stokes equation):

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}.\nabla\mathbf{v}\right) = -\nabla p + \rho\mathbf{g} + \nabla.\tau$$
$$(1)\quad(2)\qquad\qquad(3)\quad(4)\quad(5)$$

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Reynolds number $Re = \frac{\text{Inertial forces}}{\text{Viscous forces}}$

Froude number $Fr^2 = \frac{\text{Inertial forces}}{\text{Gravity forces}}$

Strouhal number $St = \frac{\text{Non-stationary acceleration}}{\text{Convective acceleration}}$

Euler number $Eu = \frac{\text{Streamwise pressure variation}}{\text{Inertial forces}}$
for incompressible pipe/channel flows

Number	Definition	Meaning	Use
Reynolds	$Re = \frac{\rho U d}{\mu}$	$\frac{\text{Inertial forces}}{\text{Viscous forces}}$	
Froude	$Fr = \frac{U}{\sqrt{gd}}$	$\frac{\text{Inertial forces}}{\text{Gravity forces}}$	free surface flow
Mach	$M = \frac{U}{c}$	$\frac{\text{flow velocity}}{\text{sound velocity}}$	compressible flow
Strouhal	$St = \frac{fd}{U}$	$\frac{\text{Non-stationary acceleration}}{\text{Convective acceleration}}$	non-stationary flow
Weber	$We = \frac{\rho U^2 d}{\sigma}$	$\frac{\text{Inertia}}{\text{surface tension}}$	two-phase flow
Taylor	$Ta = \frac{\Omega^2 d^4}{\nu^2}$	$\frac{\text{Inertial forces due to rotation}}{\text{Viscous forces}}$	rotating flows
Grashof	$Gr = \frac{g \beta \nabla T d^3}{\nu^2}$	$\frac{\text{Buoyancy forces}}{\text{viscous forces}}$	natural convection
Prandtl	$Pr = \frac{\nu}{\alpha} = \frac{\mu c_p}{k}$	$\frac{\text{viscous diffusion}}{\text{thermal diffusion}}$	fluid property, for heated flow
Schmidt	$Sc = \frac{\nu}{D}$	$\frac{\text{viscous diffusion}}{\text{molecular diffusion}}$	fluid property, for mass transfer flow
Nusselt	$Nu = \frac{hl}{k}$	$\frac{\text{Convective heat transfer}}{\text{Conductive heat transfer}}$	forced convection flow

Let us consider a Newtonian **incompressible fluid** with reference parameters: t_o, l_o, v_o, ρ_o, p_o, μ_o, g_o for, respectively time, length, velocity, density, pressure, viscosity and gravity scales.

Reduced variables:

$$t_* = \frac{t}{t_o} \;,\; x_* = \frac{x}{l_o} \;,\; y_* = \frac{y}{l_o} \;,\; z_* = \frac{z}{l_o}$$

$$\mathbf{v}_* = \frac{\mathbf{v}}{v_o} \;,\; p_* = \frac{p}{p_o} \;,\; \mathbf{g}_* = \frac{\mathbf{g}}{g_o}$$

Navier-Stokes equations:

$$\nabla.\mathbf{v} = 0 \qquad (1)$$

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}.\nabla\mathbf{v}\right) = -\nabla p + \mu\nabla^2\mathbf{v} + \rho\mathbf{g} \qquad (2)$$

Operators:

$$\nabla = \frac{1}{l_o}\left(\mathbf{e_1}\frac{\partial}{\partial x_*} + \mathbf{e_2}\frac{\partial}{\partial y_*} + \mathbf{e_3}\frac{\partial}{\partial z_*}\right) = \frac{1}{l_o}\nabla_*$$

$$\nabla^2 = \frac{1}{l_o^2}\left(\frac{\partial^2}{\partial x_*^2} + \frac{\partial^2}{\partial y_*^2} + \frac{\partial^2}{\partial z_*^2}\right) = \frac{1}{l_o^2}\nabla_*^2$$

$$\frac{\partial}{\partial t} = \frac{1}{t_o}\frac{\partial}{\partial t_*}$$

$$\mathbf{v}.\nabla\mathbf{v} = \frac{v_o^2}{l_o}\mathbf{v}_*.\nabla_*\mathbf{v}_*$$

$$(1) \Rightarrow \left(\frac{v_o}{l_o}\right) \nabla_* . \mathbf{v}_* = 0 \Rightarrow \qquad \boxed{\nabla_* . \mathbf{v}_* = 0} \qquad \text{Mass}$$

$$(2) \Rightarrow \frac{\rho_o v_o}{t_o} \frac{\partial \mathbf{v}_*}{\partial t_*} + \frac{\rho_o v_o^2}{l_o} \mathbf{v}_* . \nabla \mathbf{v}_* = -\frac{p_o}{l_o} \nabla_* p_* + \frac{\mu_o v_o}{l_o^2} \mu_* \nabla_*^2 \mathbf{v}_* + \rho_o g_o \mathbf{g}_*$$

$$\left(\frac{f_o l_o}{v_o}\right) \frac{\partial \mathbf{v}_*}{\partial t_*} + \mathbf{v}_* . \nabla_* \mathbf{v}_* = -\left(\frac{p_o}{\rho_o v_o^2}\right) \nabla_* p_* + \left(\frac{\mu_o}{\rho_o v_o l_o}\right) \mu_* \nabla_*^2 \mathbf{v}_* + \left(\frac{g_o l_o}{v_o^2}\right) \mathbf{g}_*$$

Dimensionless numbers appear: $St = \frac{f_o l_o}{v_o}$, $Eu = \frac{p_o}{\rho_o v_o^2}$, $Re = \frac{\rho_o v_o l_o}{\mu_o}$, $Fr = \frac{v_o^2}{g_o l_o}$

$$\boxed{St \frac{\partial \mathbf{v}_*}{\partial t_*} + \mathbf{v}_* . \nabla_* \mathbf{v}_* = -Eu \nabla_* p_* + \frac{1}{Re} \mu_* \nabla_*^2 \mathbf{v}_* + \frac{1}{Fr} \mathbf{g}_*} \qquad \text{Momentum}$$

If we choose $p_o = \rho_o v_o^2$ as pressure reference:

$$St \frac{\partial \mathbf{v}_*}{\partial t_*} + \mathbf{v}_* . \nabla_* \mathbf{v}_* = -\nabla_* p_* + \frac{1}{Re} \mu_* \nabla_*^2 \mathbf{v}_* + \frac{1}{Fr} \mathbf{g}_*$$

For a stationary flow:

$$\mathbf{v}_* . \nabla_* \mathbf{v}_* = -\nabla_* p_* + \frac{1}{Re} \mu_* \nabla_*^2 \mathbf{v}_* + \frac{1}{Fr} \mathbf{g}_*$$

For **compressible flows**, if fluid is assimilated to an ideal gas:

Mass

$$St\frac{\partial \rho_*}{\partial t_*} + \nabla_*.\rho_* v_* = 0$$

Momentum

$$St\rho_*\frac{\partial \mathbf{v}_*}{\partial t_*} + \rho_*\mathbf{v}_*.\nabla\mathbf{v}_* = -\frac{1}{\gamma M^2}\nabla_* p_* + \frac{1}{Re}\left[\mu_*\nabla_*^2\mathbf{v}_* + \frac{\mu_*}{3}\nabla_*\left(\nabla_*.\mathbf{v}_*\right)\right] + \frac{1}{Fr}\rho_*\mathbf{g}_*$$

Energy

$$\rho_* c_{p*}\left[St\frac{\partial T_*}{\partial t_*} + \mathbf{v}_*.\nabla_* T_*\right] = \frac{1}{RePr}\nabla_*.\left(k_*\nabla_* T_*\right) - \frac{\gamma-1}{\gamma}p_*\nabla_*.\mathbf{v}_* + (\gamma-1)\frac{M^2}{Re}\phi_{v*}$$

where: ϕ_v is the viscous dissipation function, T is the temperature and γ is the heat capacity ratio.

Outline

"I found that I was fitted for nothing so well as the study of Truth, as having a nimble mind and versatile enough to catch the resemblance of things (which is the chief point), and at the same time steady enough to fix and distinguish their subtle differences."

Francis Bacon (1561-1628)

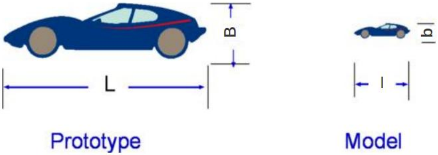
Relationship between the model and the prototype in testing?

1. Geometric similarity
2. Kinematic similarity
3. Dynamic similarity

$\Rightarrow$ **Complete similitude**

Geometric similarity

Two geometrical objects are called **similar** if they both have the same shape:
all linear length scales of one protoype are a fixed ratio of all corresponding length scales of the model and all angles are preserved.



$$\frac{l}{L} = \alpha, \ \frac{b}{B} = \alpha, \ \frac{B}{L} = \frac{b}{l}$$

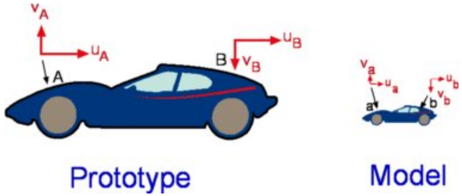
But, in some cases it is **not possible** to have a **complete geometric similarity**!!

⇒ hydraulic models are sometimes **distorted** with **different** scale ratios in horizontal and vertical directions.

Watch out! Scale decrease may create new phenomena!

(for example, capillarity, surface tension...)

Kinematic similarity



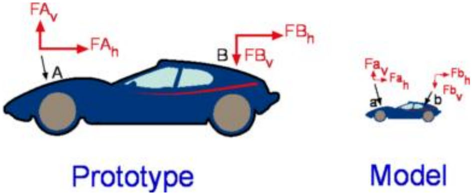
This similarity requires that the **length and time scales** be similar between the model and the prototype implying that **velocities** at corresponding points be similar.

⇒ Then fluid **streamlines** are **similar**

$$\frac{v_A}{u_A} = \frac{v_a}{u_a}, \; \frac{v_B}{u_B} = \frac{v_b}{u_b}, \; \ldots$$

Remark: **Kinematic** *similarity includes* **geometric** *similarity.*

Dynamic similarity



Two geometrically similar objects are said to be **dynamically similar** if the **forces** acting at corresponding locations on the two objects are everywhere in the same ratio.

$$\frac{FA_v}{FA_h} = \frac{Fa_v}{Fa_h}, \ \frac{FB_v}{FB_h} = \frac{Fb_v}{Fb_h}, \ \ldots .$$

Of the three similarities, the dynamic similarity is the most **restrictive**.

To achieve **dynamic similarity**, all **dimensionless numbers** relevant to the flow must be **preserved** between the model and the protoype:

$$(\pi_1)_{model} = (\pi_1)_{prototype}, \ (\pi_2)_{model} = (\pi_2)_{prototype}, \ \ldots$$

Similarity conditions for incompressible flows

Dimensionless numbers: Reynolds, Froude

- When a flow has **no free surface** (for example: internal pipe flows), gravity forces may be included to pressure forces in movement equations as the hydraulic charge: $p_o = p + \rho g z$ (in replacement of the pressure). Then **it is no more necessary to use Froude number** in similarity conditions.
- When a flow has **a free surface with a changing position**, we can not include gravity forces in the hydraulic charge, **Reynolds and Froude numbers have also to be preserved** to achieve dynamic similarity.
- If **surface tension** intervenes in studied phenomena, then **Weber number must be preserved** with Reynolds and Froude numbers.

Similarity conditions for compressible flows

Dimensionless numbers: Reynolds, Mach

- Gravity forces can be neglected so **Froude number is not necessary** to achieve dynamic similarity (in the case of great scale atmospheric flows, gravity forces must be taken into account).
- In practice, it is difficult to preserved Mach and Reynolds numbers for reasonable size of models. So, **when Mach number is small**, its effects are theoretically calculated and we choose **only Reynolds number to conserve dynamic similarity**.
- On the other hand, for a **great Mach number**, compressibility effects are preponderant then **Mach number must be conserved and Reynolds number must have a realistic value**. As the model size reduction decreases Reynolds number, we have to perform experiments with a higher pressure or a weaker temperature in order to have a valid Reynolds number to operate: using **pressurized** or **cryogenic** wind tunnel is necessary in transsonic flow studies.

Outline

1. International System of Units
   - History
   - Base and derived units
2. Dimensional Analysis
   - Vaschy-Buckingham Theorem (π-Theorem)
   - Dimensional Analysis of balance equations
   - Principal dimensionless numbers
   - Dimensional Analysis of balance equations
3. Similarity
   - Model geometry
   - Similarity conditions
4. Conlusion

Dimensional analysis is important to:

- Find physical laws
- Simplify problems
- Detect relevant parameters
- Build a model

In order to achieve this analysis, it is important to master:

- Vaschy-Buckingham theorem (π-Theorem)
- Know dimensionless numbers
- Know similarity conditions rules