# Dimensional Analysis and Similarity

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## 1 Studying a cargo-ship with a scale model

In order to determine the forward resistance of a cargo-ship, a model is built to perform measurements in a test pool. The characteristics of the studied cargo-ship are presented in the Table 1.

Figure 1: A cargo-ship (Pavel Korchagin)

| | |
|---|---|
| Waterline length $L_1$ | 180 $m$ |
| Width $B_1$ | 24.9 $m$ |
| Draft $D_1$ | 10.8 $m$ |
| Wetted surface $S_1$ | 3 600 $m^2$ |

Table 1: Cargo-ship characteristics

**1.** On which quantities does the total resistance of the cargo-ship $R$ depend ?

**2.** Considering $\rho$ the water density, $L_1$ the waterline length and $V_1$ the cargo-ship velocity, express the problem dimensionless groups.

**3.** The total resistance to forward of the ship can be divided into four components:

- $R_f$: Frictional resistance due to the water friction on the hull of the ship (related to fluid viscosity)
- $R_v$: Backwash resistance due to vortices created in the detachment zone

- $R_w$: Wave resistance
- $R_a$: Aerodynamic resistance (linked to the emerged part of the ship)

In a first approximation, $R_v$ and $R_w$ can be grouped in the direct resistance $R_d$: $R_d = R_v + R_w$, which characterizes the resistance opposed by waves induced by the ship on water surface. As the aerodynamic resistance $R_a$ is negligible against the others, total resistance can be divided into two independent forces:

$$R = R_f + R_d$$

Each dimensionless force $\frac{R_f}{\rho L_1^2 V_1^2}$ and $\frac{R_d}{\rho L_1^2 V_1^2}$ depends only on one dimensionless number:

Can you propose on which parameter the frictional resistance depends? Can you propose on which parameter the direct resistance depends? What is the new expression for the quantity $\frac{R}{\rho L_1^2 V_1^2}$?

**4.** A model is built to represent this cargo-ship. The considered fluid is water ($\rho = 1000\ kg.m^{-3}$ and $\mu = 10^{-3}\ kg.m^{-1}.s^{-1}$).

Is the similarity between the prototype and the model complete?

Is the evaluation of the frictional resistance possible?

**5.** Model length $L_2$ is equal to 6 $m$. What is its scale?

We want to determine the total resistance as a function of the the cargo-ship velocity. Pool tests were performed with a model velocity $V_2 = 1.61\ m.s^{-1}$.

What is the value of the Reynolds number $Re_{L_2}$ for the model? Is the hypothesis for neglecting aerodynamic resitance valid?

What is the prototype velocity $V_1$ associated with this experiment?

**6.** At $V_2 = 1.61\ m.s^{-1}$, the measured total resistance is: $R_2 = 20\ N$.

Frictional resistance can be written as:

$$R_f = \frac{1}{2} C_f \rho S V^2$$

with: $C_f = \frac{0.074}{Re^{0.2}}$ for $Re_L < 10^7$ (Prandtl)
$C_f = \frac{0.455}{(\log_{10} Re)^{2.584}}$ for $Re_L > 10^7$ (Prandtl-Schlichting)

What is the expression of the model frictional resistance $R_{f2}$?

**7.** What is the expression of the cargo-ship total resistance $R_1$?

**8.** What is the required propeller power enabling the cargo-ship to navigate?

## 2 The atomic explosion of 1945

"Two years ago some motion picture records by Mack (1947) of the first atomic explosion in New Mexico were declassified. These pictures show not only the shape of the luminous globe which rapidly spread out from the detonation centre, but also gave the time, t, of each exposure after the instant of initiation. On each series of photographs a scale is also marked so that the rate of expansion of the globe, or 'ball of fire', can be found."

Sir Geoffrey Taylor, *The formation of a blast wave by a very intense explosion. II. The atomic explosion of 1945*, Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences, Vol. **201**, No. 1065. (Mar. 22, 1950), pp. 175-186.

Details of the strength of the first atomic bomb in 1945 were classified until the 1960s. However, the British physicist G.I. Taylor was able to give a very accurate estimate of power released by the

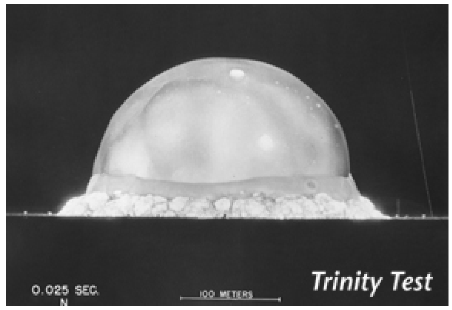


Figure 2: One picture recorded by J.E. Mack of the first atomic explosion (Trinity test)

explosion from photographs of the cloud expansion. The explosion induced an expanding spherical fireball whose edge corresponds to a powerful shock wave. The method relies on the asumption that the solution of the problem of this expanding blast-wave is auto-similar, and consists of a measure of the radius $R$ *vs.* time from the pictures.

**1.** The theory is exposed in this article:

Sir Geoffrey Taylor, *The formation of a blast wave by a very intense explosion. I. Theoretical discussion*, Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences, Vol. **201**, No. 1065. (Mar. 22, 1950), pp. 159-174.

The radius of the blast wave $R$ depends on: the released energy $E$, the time $t$, the gas density $\rho$ and the ratio of specific heats $\gamma$.

**2.** What are the dimensionless numbers of the problem?

**3.** Give an expression of the released energy as a function of the dimensionless parameters.

**4.** From the photographs used by Taylor and presented in the figure 4, check that the solution is auto-similar.

**5.** Assuming that the dimensionless constants are of order unity, calculate the released energy $E$ at time $t = 62\ ms$, when the radius $R$ is equal to 185 $m$ (air density will be taken at $20\,^\circ C$, $\rho = 1.2 kg.m^{-3}$).

Knowing that explosion energy released is generally expressed in megatons of TNT (equivalent energy of an explosion of one million tons of TNT) and that: $1Mt$ of TNT $= 4.184\ 10^{15} J$, what is the energy of Trinity test in equivalent tons of TNT?

Compare with other famous explosions:

| Test name | Location | Released energy (Mt of TNT) |
|---|---|---|
| *Little boy* | Hiroshima (1945) | 0.015 |
| *Fat man* | Nagasaki (1945) | 0.022 |
| *Tsar Bomba* | Nouvelle Zemble (1961) | 50 |
| *Licorne* | Mururoa (1968) | 1 |
| | Mururoa (1995) | 0.020 |

Table 2: Examples of atomic bomb explosions

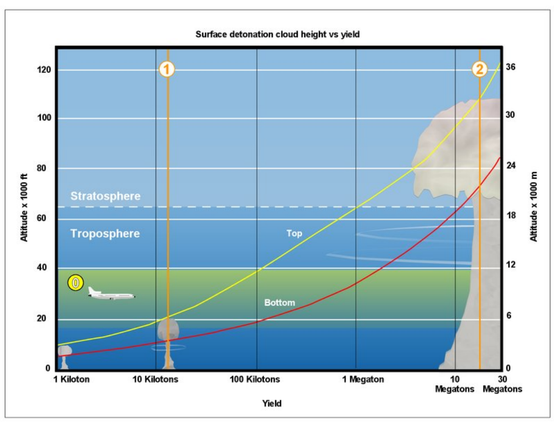


Figure 3: Evolution of surface detonation cloud height according to released energy

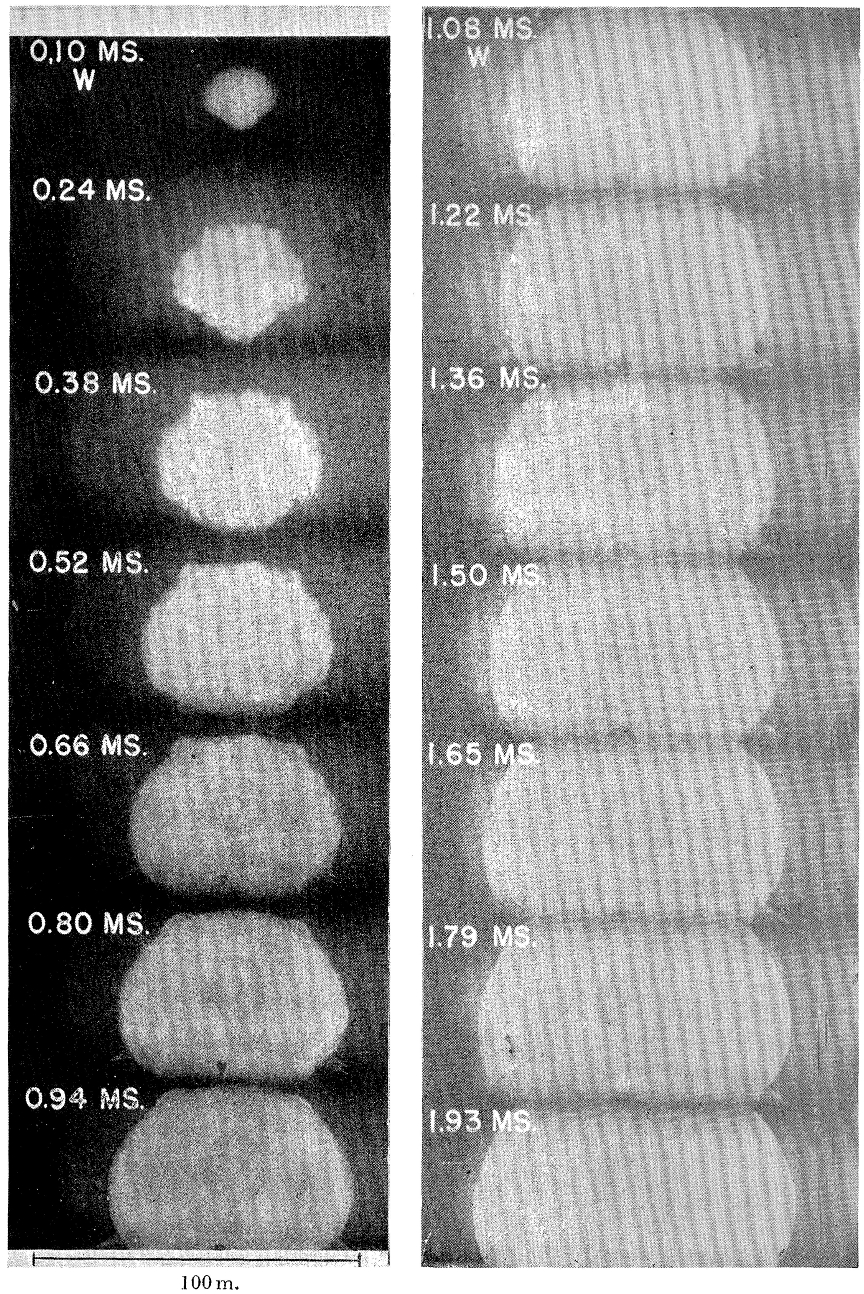


FIGURE 6. Succession of photographs of the 'ball of fire' from $t = 0{\cdot}10$ msec. to $1{\cdot}93$ msec.

Figure 4: Trinity test photographs