Sensors and measurements in fluid mechanics

A. Danlos[a], F. Ravelet[a]
[a] *Arts et Metiers ParisTech, DynFluid,*
151 boulevard de l'Hôpital, 75013 Paris, France.
contact: florent.ravelet@ensam.eu

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1 Flow rate measurement with inclined venturi

Using a venturi-meter is a cost effective method to measure flow rate.
Considering a 1D non viscous stationary flow between areas A_0 and A_1, it is possible to measure the flow rate for any inclination angle of the system.
The studied fluid is incompressible and its volumetric density is $\rho = 820\ kg.m^{-3}$. The venturi inlet area A_0 is characterized by its diameter $D_0 = 125\ mm$. The second pressure measurement is performed in the venturi throat, at the area A_1 of diameter $D_1 = 50\ mm$. Differential pressure is measured in a U-shaped tube filled with mercury which has a density $\rho_m = 13600\ kg.m^{-3}$. The studied venturi system is presented on figure 1.

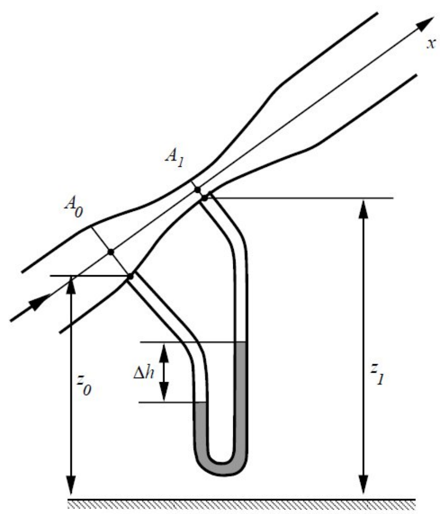

Figure 1: Inclined venturi

1. Write the Bernoulli's equation.

2. Write the pressure in each arm of the U-tube.

3. Express the height Δh.

4. Show that the volumetric flow rate is independant of the venturi inclination.

5. Calculate the volumetric flow rate.

2 Couette viscometer

Couette device is used to measure liquid viscosity. This system consists of two concentric cylinder. The internal fixed cylinder has a radius $R_1 = 5\ cm$. It is attached at its top with a torque cable. The external cylinder has a radius $R_2 = 5.02cm$ and has a uniform rotative motion with a velocity $N = 90\ tr.min^{-1}$. The system is filled with an oil until the height $h = 20\ cm$. We want to determine oil viscosity from the torque force applied by the fluid in motion on the torque cable $C = 11\ N.m$ (figure 2).

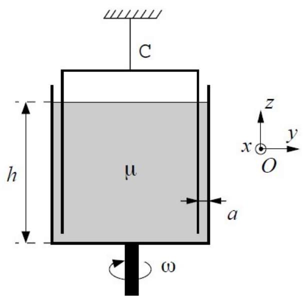

Figure 2: Couette viscometer

We assume that the flow is established, laminar, stationary and incompressible. Fluid viscosity μ to determine is constant. We can consider the oil as a Newtonian fluid with a density $\rho = 1260\ kg.m^{-3}$. Frictions at the bottom of the viscometer are neglected. Distance between internal and external cylinders at the bottom is $\delta = 1\ mm$. We have also: $R_2 - R_1 = a$.

1. With geometrical considerations, show that the flow structure between the two cylinders can be simplied in a one dimensional flow.

2. Determine velocity profile using Navier-Stokes equations and problem assumptions.

3. Deduce the expression of the tangential stress τ_0 exerted by the fluid on a surface element of the internal cylinder lateral wall.

4. Determine an expression of the torque C exerted by the fluid on the internal cylinder.

5. Deduce the value of fluid viscosity.

3 Falling drop measurement

Surface tension is a property of a wetting liquid surface enabling it to resist to an external force. The cohesive forces, among the liquid molecules, are responsible for the surface tension, which has the dimension of force per unit length, or of energy per unit area.
We will study one measurement method of a fluid surface tension: the falling drop technique.

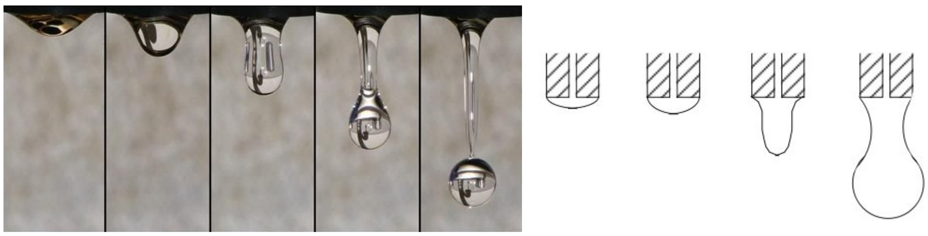

Figure 3: Falling drop movement

The liquid, whose surface tension is unknown, flows through a vertical capillary tube. Drops, with a mass m, are created at the capillary end. A drop grows, a throttling is formed, then the drop breaks at the throttling and falls leaving a meniscus at the capilary tube outlet.
Just before the drop falls, the equilibrium state is achieved. The volume of liquid situated below the throat is submitted to :

- its apparent weight (corrected by the buoyancy resulting from the integration of pressure forces on its surface).
- its surface tension γ acting on circumference.
- the Laplace overpressure due to the two radii of curvature in the throat: $\Delta p = \gamma \left(\frac{1}{r} - \frac{1}{R}\right)$ exerting on the surface πr^2 (figure 4).

where r is the throat circle radius, and R the curvature radius in the meridian plane.

1. Express all forces acting on the drop.

2. Express the surface tension of the drop.

3. Assuming $R >> r$, rewrite the surface tension expression.

4. Considering that the surrounding gas is air, simplify the previous expression.

5. In order to take into account the fraction of liquid remaining on the capillary tube, a geometrical correction factor f is introduced in the calculation of the surface tension:

$$\gamma = \frac{mg}{\pi f a}$$

where a is the external radius of the capillary tube.

The f factor depends on the aspect ratio $a/V^{1/3}$, where V is the drop volume (Table 1).

Determine the surface tension for an oil in the following case:
Its relative density is 0.98 for a temperature $T = 22\,^\circ C$. At this temperature, the water density is $\rho_{water} = 997\ kg.m^{-3}$. The capillary tube has a capacity of 2 mL and its external diameter is $a = 0.8\ mm$ and its internal diameter is $d_i = 0.5\ mm$. Surface tension measurement used 200 drops to obtained a mass of liquid of $M_{total} = 1.094\ g$.

$a/V^{1/3}$	0.30	0.35	0.40	0.45	0.50	0.55	0.60
f	1.4512	1.4022	1.3656	1.3258	1.3030	1.2724	1.2500
$a/V^{1/3}$	0.65	0.70	0.75	0.80	0.85	0.90	0.95
f	1.2342	1.2186	1.2064	1.2000	1.1984	1.1996	1.2068

Table 1: Factor f values according to the aspect ratio

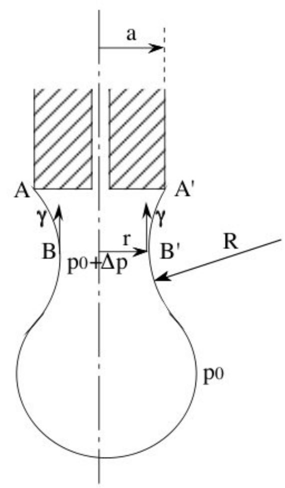

Figure 4: Details of a falling drop